Stat 98/198 – Spring 09 – Exam P DeCal – HW #6
Due 3/12

79. A device contains two components. The device fails if either component fails. The joint density function of the lifetimes of the components, measured in hours, is $f(s,t)$, where $0 < s < 1$ and $0 < t < 1$.

What is the probability that the device fails during the first half hour of operation?

(A) $\int_0^{0.5}\int_0^{0.5} f(s,t)\,ds\,dt$ (B) $\int_0^{1}\int_0^{0.5} f(s,t)\,ds\,dt$ (C) $\int_{0.5}^{1}\int_{0.5}^{1} f(s,t)\,ds\,dt$

(D) $\int_0^{0.5}\int_0^{1} f(s,t)\,ds\,dt + \int_0^{1}\int_0^{0.5} f(s,t)\,ds\,dt$ (E) $\int_0^{0.5}\int_{0.5}^{1} f(s,t)\,ds\,dt + \int_0^{1}\int_0^{0.5} f(s,t)\,ds\,dt$

88. The waiting time for the first claim from a good driver and the waiting time for the first claim from a bad driver are independent and follow exponential distributions with means 6 years and 3 years, respectively.

What is the probability that the first claim from a good driver will be filed within 3 years and the first claim from a bad driver will be filed within 2 years?

(A) $\frac{1}{18}\left(1-e^{-2/3}-e^{-1/2}+e^{-7/6}\right)$ (B) $\frac{1}{18}e^{-7/6}$ (C) $1-e^{-2/3}-e^{-1/2}+e^{-7/6}$

(D) $1-e^{-2/3}-e^{-1/2}+e^{-1/3}$ (E) $1-\frac{1}{3}e^{-2/3}-\frac{1}{6}e^{-1/2}+\frac{1}{18}e^{-7/6}$

89. The future lifetimes (in months) of two components of a machine have the following joint density function:

$$f(x,y)=\begin{cases}\dfrac{6}{125,000}(50-x-y) & \text{for } 0<x<50-y<50\\ 0 & \text{otherwise.}\end{cases}$$

What is the probability that both components are still functioning 20 months from now?

(A) $\dfrac{6}{125,000}\displaystyle\int_0^{20}\int_0^{20}(50-x-y)dydx$ (B) $\dfrac{6}{125,000}\displaystyle\int_{20}^{30}\int_{20}^{50-x}(50-x-y)dydx$

(C) $\dfrac{6}{125,000}\displaystyle\int_{20}^{30}\int_{20}^{50-x-y}(50-x-y)dydx$ (D) $\dfrac{6}{125,000}\displaystyle\int_{20}^{50}\int_{20}^{50-x}(50-x-y)dydx$

(E) $\dfrac{6}{125,000}\displaystyle\int_{20}^{50}\int_{20}^{50-x-y}(50-x-y)dydx$

90. An insurance company sells two types of auto insurance policies: Basic and Deluxe. The time until the next Basic Policy claim is an exponential random variable with mean two days. The time until the next Deluxe Policy claim is an independent exponential random variable with mean three days.

What is the probability that the next claim will be a Deluxe Policy claim?

(A) 0.172 (B) 0.223 (C) 0.400 (D) 0.487 (E) 0.500

91. An insurance company insures a large number of drivers. Let X be the random variable representing the company's losses under collision insurance, and let Y represent the company's losses under liability insurance. X and Y have joint density function

$$f(x,y)=\begin{cases}\dfrac{2x+2-y}{4} & \text{for } 0<x<1 \text{ and } 0<y<2\\ 0 & \text{otherwise.}\end{cases}$$

What is the probability that the total loss is at least 1 ?

(A) 0.33 (B) 0.38 (C) 0.41 (D) 0.71 (E) 0.75